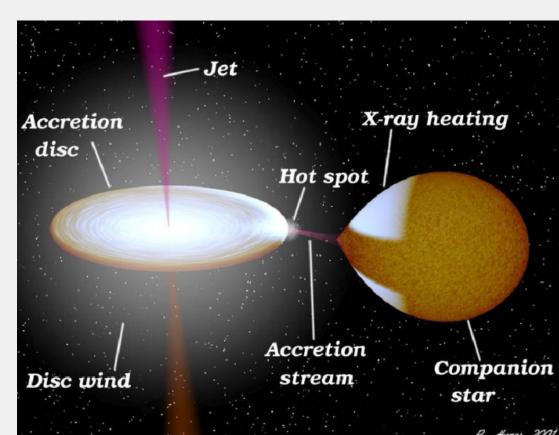
### Accretion onto the NS

- Neutron stars (NS) in binary systems accrete matter from their companion stars, such as in Low-Mass X-ray Binaries (LMXBs), where the companion is a low-mass star.
- Matter from a companion star forms a disc before falling onto the neutron star, releasing gravitational energy and producing high-energy radiation, primarily as X-rays.
- In the spectra of such accreting NSs, different features are observed depending upon the properties of the system such as rate of accretion and NS magnetic field.



Representation of a Low Mass Xray Binary, Source: CESAR ESA

## Accretion disk model around a NS

• To include the effect of the flux from the neutron star boundary layer, we solve the following three equations:

Energy Balance equation

$$Q_{heat} - Q_{cool} - Q_{adv} = 0 (1)$$

where 
$$Q_{heat} = \frac{\dot{M}}{4\pi r} (\Omega r^2 - j\Omega_{NS}^K R_{NS}) \frac{d\Omega}{dr}$$
,  $Q_{cool} = \frac{4\sigma_{bol}}{3\kappa} \frac{T^4}{\rho H}$ ,  $Q_{adv} = \frac{\dot{M}}{4\pi r^2} \frac{P}{\rho}$ 

Equation for Sub-Keplerian angular velocity of the disk and radial pressure gradient.

$$\frac{d\Omega}{dr} = \frac{v_r}{vr^2} (\Omega r^2 - j\Omega_{NS}^K R_{NS}^2) \; ; \quad \frac{dP}{dr} = (\Omega^2 - \Omega_k^2) \rho r \tag{2}$$

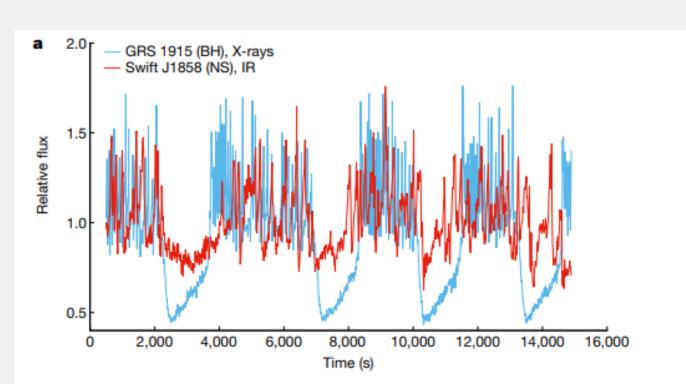
where 'j' being the external parameter and  $\Omega_{NS}^{K} = \sqrt{\frac{GM_{NS}}{R_{NS}^{3}}}$ , is the Keplerian velocity of the NS.

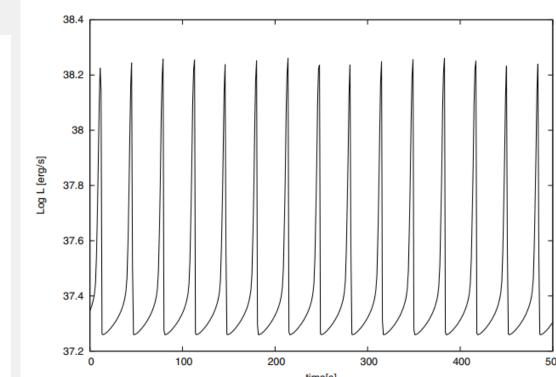
- For the non-Keplerian regime, these equations from Popham and Sunyaev (2001) are used to solve for temperature (T), angular velocity ( $\Omega$ ) and height (H).
- Further, we use the equation of state to obtain pressure (P) and vertically averaged hydrostatic equilibrium density  $(\rho)$

$$P = \frac{4\sigma_{bol}}{3c}T^4 + \frac{K_b}{m_H}\rho T \; ; \; \frac{P}{\rho} = C_3\Omega_k^2 H^2$$
 (3)

### Instabilities in NS and BH accretion disks

- Radiation pressure instabilities (RPIs) are thermoviscous instabilities from inner portions of accretion disks, causing short-term variations in the X-ray emission, on timescales of seconds.
- Janiuk et al. (2002) model explained these variabilities due to RPIs in black hole X-ray binaries, such as GRS 1915-105.
- Recently, Vincentelli (2023) demonstrated that neutron stars exhibit similar accretion disk instabilities, as shown in the figure.





Left: Short-term variabilities in SWIFT-J1858.6-0814, Source: Vincentelli (2023) Right: Instabilities obtained by Janiuk et al. (2015) using GLADIS code to model black hole accretion disk instability for microquasar IGR J17091-3624

# Sub-Keplerian $\Omega$ profile and NS boundary $R_B$

• To solve for boundary layer, we take an analytical form of sub-Keplerian angualr velocity

$$\Omega^{2}(r) = \Omega_{NS}^{2} - \frac{GM_{NS}}{r^{4}}(R_{BL} - \frac{4}{3}r) + \frac{GM_{NS}}{R_{NS}^{4}}(R_{BL} - \frac{4}{3}R_{NS})$$
(4)

according to the physical boundary values and nature of  $\frac{d\Omega}{dr}$  in Eq(2).

• Then, we use the relation  $\frac{d\Omega}{dr} = 0$  at  $R = R_{BL}$ , as

$$\Omega_{BL}R_{BL}^2 = j\Omega_{NS}^K R_{NS}^2 \tag{5}$$

"j' plays the role of ratio of keplerian and sub-keplerian angular momentum.

• Further, using averaged form of  $\frac{dP}{dr}$  from eq(2) and  $\frac{dP}{dh}$  from eq(3), we can get height of the disk at  $R = R_{BL}$ , as

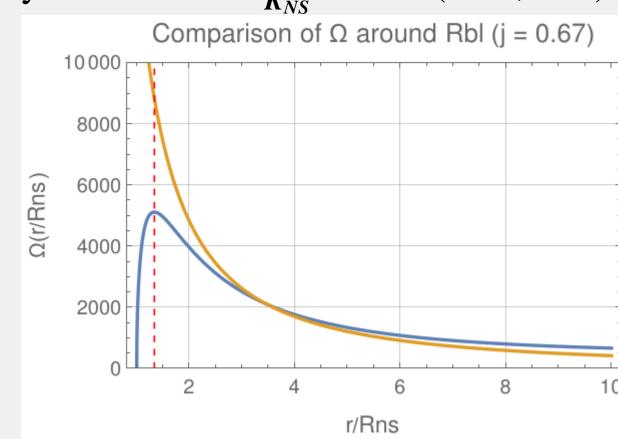
$$H = \left(\sqrt{1 - \left(\frac{\Omega_{BL}}{\Omega_{BL}^{K}}\right)^{2}}\right) R_{BL}$$
 (6)

where  $\Omega_{BL}$  is sub-keplerian angular velocity at  $R = R_{BL}$ ,

• Although, the outer regions of the disk can still behave in keplerian way, these quantities offer a promising constraint on the inner regions of the disk, around  $R_{BL}$ .

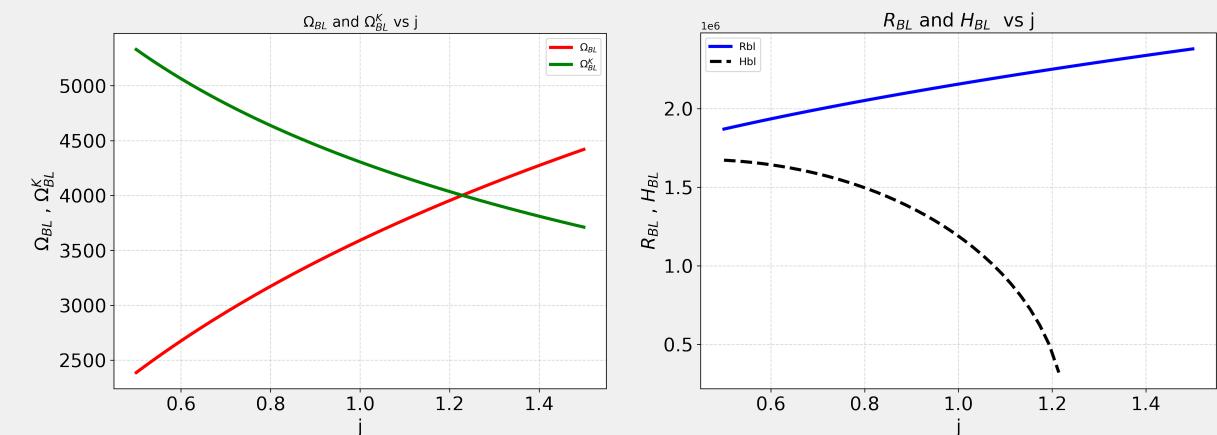
## **Analytical Results**

• For typical NS parameters,  $M_{NS} = 1.4 M_{\odot}$ ,  $R_{NS} = 10^6 \, \mathrm{cm}$ ,  $\Omega_{NS} = 100$ , j = 0.67. we get a physical value of  $\frac{R_{BL}}{R_{NS}}$  within (1.30,1.50).



Variation of  $\Omega(r)$  and  $\Omega_K(r)$  around the boundary layer, for  $R_{BL}=1.338R_{NS}$ 

• Further, we check their variations with "j" which is the ratio of the angular momentum accretion rate in the boundary layer, to that in the thin disk.



Left: Variation of Keplerian and Sub-keplerian  $\Omega$  with respect to "j", • Importantly, we note that the above results allow us to set a constraint on the range of "j" i.e. 0.66 < j < 1.24.

Right: Variation of boundary radius  $R_{BL}$  and disk height  $H_{BL}$  with respect to "j"

## **Summary and Further Work**

- Our goal is to obtain the profile of sub-Keplerian boundary layer in the vicinity of a NS, to explain the RPI instabilities in the inner regions of the disk.
- Starting with a possible analytical form of sub-Keplerian  $\Omega(r)$ , we obtain reasonable values of  $\Omega_{BL}$ ,  $R_{BL}$  and  $H_{BL}$  and put a constraint on the angular momentum ratio parameter "j".
- We need to obtain solutions for other physical quantities such as temperature, pressure, density, radial velocity and optical depth. An important factor to be considered is the optical thickness around boundary radius, which affects the dominant energy term in energy balance equation. Proper description of radiative processes in the optically thin and thick regime is essential.

#### References

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